The Farmer

Let g_i be a field or a strip. Denote by n_i the number of cypresses in a field or a strip. If we denote by e_i the number of olive trees in a g_i, we have: $e_i = n_i$ if g_i is a field or $e_i = n_i - 1$ if g_i is a strip.

Consider now the following KNAPSACK problem: max $\sum_{i=1}^{n+m} e_i x_i$, such that $\sum_{i=1}^{n+m} n_i x_i \leq Q$ and $x_i \in \{0, 1\}$, where n,m are the numbers of fields and strips respectively.

An optimum solution x_i^*, $1 \leq i \leq n + m$, of this problem consists of a subset of g_i such that the total number of their cypresses is at most Q and the total number of the included olive trees is maximized. However in general $Q' = \sum_{i=1}^{n+m} n_i x_i < Q$. If this is the case, then there is some g_i such that $x_i^* = 0$ and $n_i > Q - Q'$, for otherwise the optimum solution can be improved by the inclusion of g_i, a contradiction. Therefore, adding a chain of $Q - Q'$ cypresses of g_i and its $Q - Q' - 1$ olive trees to the optimum solution of the knapsack problem above, yields to an optimum solution. The KNAPSACK problem can be solved optimally in $O\big((n + m)Q\big)$ time by a Dynamic Programming algorithm.